Problem 1.2

An accelerometer indicates that a structure is vibrating harmonically at 82 cps with a maximum acceleration of 50 g. Determine the amplitude of vibration.

[TYPO: 50 g is read as 50 grams. It should be written as $50g$.]

Solution

An object in harmonic motion has a position given by

$$x(t) = A \sin 2\pi f t,$$

where A is the amplitude and f is the linear frequency. Differentiate the position with respect to time to get the velocity.

$$\dot{x}(t) = A(2\pi f) \cos 2\pi f t$$

Differentiate the velocity with respect to time to get the acceleration.

$$\ddot{x}(t) = -A(2\pi f)^2 \sin 2\pi f t$$

The maximum acceleration is

$$a_{\max} = A(2\pi f)^2.$$

Set it equal to $50g$ and $f = 82$ cycles per second and solve the equation for A.

$$A(2\pi \cdot 82)^2 = 50g$$

Therefore, noting that $g \approx 9.81$ m/s^2,

$$A = \frac{25g}{13448\pi^2} \approx 0.00185 \text{ m} = 0.185 \text{ cm}.$$